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PRELIMINARY DEVELOPMENT OF GUST DESIGN PROCEDURES BASED ON POWER SPECTRAL TECHNIQUES

VOLUME II.   Summay of Possible Procedures

JOHN C. HOUBOLT

TECHNICAL REPORT AFFDL-TR-66-58, VOL. II

AIR FORCE FLIGHT DYNAMICS LABORATORY
RESEARCH AND TECHNOLOGY DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

NOTICES

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FOREWORD

This final report was prepared by Aeronautical Research Associates of Princeton, Inc., Princeton, New Jersey, in fulfillment of USAF Contract Number AF33(615)-2144; BPSN: 5(6899-61430014-000-600-FD); Project and Task Numbers: None. This effort was done under Laboratory Director's Funds. This project is part of a program to provide the theoretical and practical applied research necessary to maintain USAF structural design criteria for aerospace flight vehicles and to establish optimum structural requirements, flight limitations and design philosophies commensurate with vehicle structural life.

The work was administered under the direction of the Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, Mr. Paul L. Hasty (FDTR), Project Engineer. This investigation was administered under Project 1367 and Task 136702; all follow-on work will be funded under this task.

The work reported in this study was conducted by Aeronautical Research Associates of Princeton, Inc. with Dr. John C. Houbolt as principal investigator, and covers the period from November 1964 to March 1966. The manuscript was released by the author in March 1966 for publication as an RTD Technical Report.

The Contractor's report number is ARAP Report No. 83, Volumes I and II.

This technical report has been reviewed and is approved.

FRANCIS J. JANIK, JR.
Chief, Theoretical Mechanics Branch
Structures Division

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ABSTRACT

A development is given of four possible procedures for design-ing aircraft for gusts based on power spectral techniques. Each is of a different level of complexity but all are attractive at the moment. In one, explicit consideration of atmospheric turbu-lence makeup is bypassed altogether. Another makes use of simple over-all composite values of r.m.s. value of turbulence intensity and proportion of time in turbulence. A third is more detailed and considers mission aspects explicitly. A fourth is a rather simple comparison approach.

Items which represent areas of uncertainty are brought out, chief among these being the turbulence scale value, and the prac-tical means for establishing cutoff values in the determination of the basic frequency parameter N_0 of the airplane. The need for a more unified and consistent data gathering program is brought out, among various agencies and countries, and from the standpoint of operational as well as research flights. Analogous rigid body treatments by spectral and discrete-gust lines are made and com-pared so that the discrete-gust design levels could be used as a guide to help establish input number by a spectral approach.

The material of the report forms the basis for the next phase of study which has the aim of developing one or more of the procedures into a firm design language. Computational data from existing gust critical aircraft will help guide number or design boundary selection in this task.

CONTENTS

ILLUSTRATIONS

SYMBOLS

a	lift curve slope
A	structural parameter; $\sigma_x = A\sigma_w$
c, c_0	wing chord
f	frequency, cps
h	altitude
$H(\omega)$	frequency response function due to sinusoidal gust
k	reduced frequency, $k = \omega_c/2V$
K_g	gust alleviation factor
L	scale of turbulence
n	total number of times load level x is crossed with positive slope
n_0	total number of zero crossings with positive slope
N	number of times per second load level x is crossed with positive slope
N_0	number of zero crossings per second with positive slope
$p(\sigma)$	probability distribution function of σ
P	proportion of time spent in turbulence
s_r	reference stress as used in fatigue
S	wing area
T	total (lifetime) flight time
T_t	total time in turbulence
T_F	average duration of a mission flight
U_d	gust velocity value similar to U_{de}
U_{de}	"derived" gust velocity
V	flight velocity
V_e	equivalent airspeed

SYMBOLS (Cont'd.)

W	airplane weight
x	increment in load or response level due to gusts
$x_{L.L.}$	load level representing limit load
x_{1-g}	1 - g load level
α, β	constants
λ	wavelength
μ_g	mass ratio or mass parameter, $2W/a\rho cgS$
μ_0	mass ration, $\mu_0 = 4\mu_g$
ρ	air density
σ	r.m.s. value
σ_c	composite r.m.s. value of vertical gust velocity
σ_x	r.m.s. value of response x
σ_w	r.m.s. value of vertical gust velocity
$\phi(\Omega)$	power spectrum
ω	angular frequency
Ω	spacial frequency, ω/V

Note:

$$\Omega = \frac{\omega}{V} = \frac{2k}{c} = \frac{2\pi}{\lambda}$$

$$\phi(\Omega) = L\phi(L\Omega) = V\phi(\omega) = \frac{c}{2}\phi(k)$$

CHAPTER 1

INTRODUCTION

Under contract with the Air Force Flight Dynamics Laboratory, Aeronautical Research Associates of Princeton, Inc. is developing procedures for designing aircraft for gust encounter based on power spectral techniques. Some of the basic theoretical consider- ations that have evolved so far in the first phase of study are presented in Volume I. Besides the treatment of gust design from a load exceedance point of view, Volume I also includes some preliminary thoughts on a new concept for treating the fatigue life of the aircraft structure as well, using a common notation and procedure.

In the present companion volume, further results of the initial phase of the study are given. In particular, a discussion is given of four possible design procedures that have been devel- oped, which range in complexity and in degree of sophistication from a simple lumped parameter approach not too unlike the past discrete gust design procedure, but which incorporate the more accurate and rational ingredients of the power spectral approach, to a more detailed mission approach which takes into account the differing gust encounter effects of various stages of flight. Some of the other theoretical work that has been developed is presented also.

Simultaneously with the present study, other related work has been carried on in some of the aircraft companies, sponsored also by Wright Field and under monitorship by Dr. J.C. Houbolt, the principal investigator of the present investigation. These

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studies were computational in nature and were aimed at determining, for existing gust critical airplanes that were designed by the discrete-gust technique, certain basic response parameters that are significant in a power spectral approach. The intent in the next phase of study is to continue the developmental work of the first phase, to incorporate in a design procedure sense the numbers that were found for the specific airplanes investigated, and to end up with a definite spectral design procedure (or several) with definite design input numbers. One of the underlying theses that is being followed in arriving at the proper or appropriate input values is that of applying the procedure in retrospect manner to proven gust critical aircraft with the concept that if they had been designed originally by the new procedure, safe designs would have also resulted.

SECTION 2

SUMMARY OF POSSIBLE DESIGN PROCEDURES

A summary of the possible design procedures that have been developed during the past year of study is presented in this section.

In Volume I, the development of a "universal" curve for load exceedances due to gust is indicated. This curve is shown in Figure 1; it applies to any response variable x of concern, such as stress, bending moment, acceleration, etc., where x is the increment in response due to gust encounter. Other parameters are: N is the number of times per second that the response level x is crossed with positive slope, N_0 the number of times per second that the response crosses the zero value with positive slope, P is the total proportion of flight time spent in turbulence, and A is the structural response parameter relating σ_x, the r.m.s. values of x, to σ_c, the r.m.s. value of all gusts encountered during flight, according to the relation

$$\sigma_x = A\sigma_c$$

The practical aspects of determining A and N_0 are outlined in Section 4.

For abscissa values greater than about 10, the curve of Figure 1 is represented well by the equation

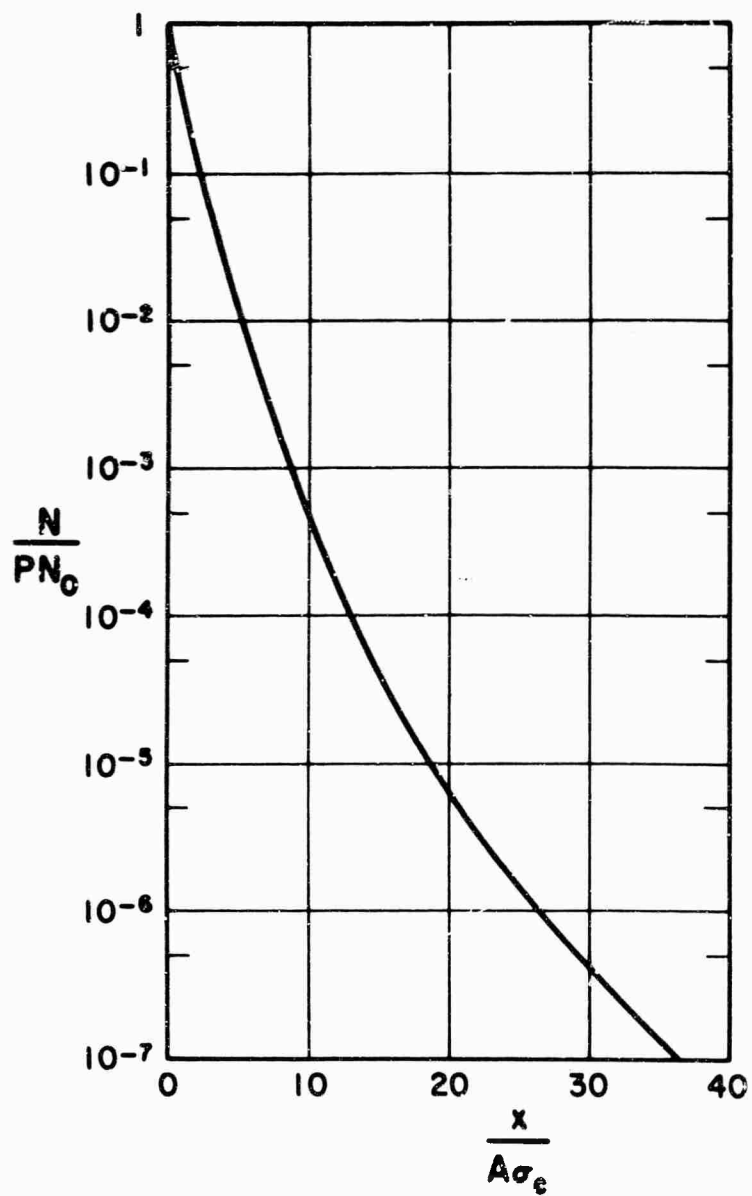$$\frac{N}{PN_0} = \left(\frac{3.36 \ A\sigma_c}{x}\right)^{6.74} \tag{1}$$

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Figure 1. "Universal" Curve for Load Exceedance.

The empirical constants 3.36 and 6.74 are tentative, being based
on limited data only; these constants are being considered further
in continuing studies and should be updated as more and more oper-
ational data are cast in the universal form suggested here. In
the lifetime $\dot{\ell}$ of flight, the total number of times a given level
x is exceeded (crossed with positive slope for x plus, or with
negative slope for x minus) follows from Equation (1) as

$$n = PTN_0\left(\frac{3.36\ A\sigma_c}{x}\right)^{6.74} \qquad (2)$$

This equation is advanced here as a master type equation for
designing aircraft for gust from a load exceedance point of view.
For design, the level x is based on chosen design values, such
as limit load or ultimate load, in accordance with the design
philosophy preferred, while n is a stipulated number denoting
the number of times that this level is allowed to be reached in
the lifetime of the airplane (on a statistical average basis). A
limit-load reference will be followed herein, such that

$$x = \Delta x = x_{L.L.} - x_{1-g}$$

where x_{1-g} is the load level in 1-g flight. For the implied
comparison of load exceedances with x on an ultimate load basis
consider

$$x_U = x_{U.L.} - x_{1-g}$$

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and assume $x_{U.L.} = 1.5 \, x_{L.L.}$ and let $x = x_L = rx_{1-g}$. Then it follows that

$$\frac{x_L}{x_U} = \frac{r}{1.5 \, r + .5}$$

and in turn from Equation (2) that

$$\frac{n_L}{n_U} = \left(\frac{1.5 \, r + .5}{r}\right)^{6.74}$$

This relation yields the example results

r or $\dfrac{x_L}{x_{1-g}}$	$\dfrac{n_L}{n_U}$
1	107
1.5	60
2	44

Thus, a single encounter of ultimate load conditions is equivalent statistically to on the order of 50 encounters of limit load. For design, it would appear then that a choice of $n = 5$ to 10 for allowable limit load exceedances represents a good margin of safety.

From Equation (2), four design procedures of varying complexity may be derived, as indicated in the following. The possibility of treating fatigue design in analogous fashion also exists, and this possibility is indicated in Volume I.

2.1. DESIGN BASED ON STIPULATED σ_c, P, AND T (Presently recommended as a second step improvement for design)

Equation (2) plots as shown in Figure 2, where the ordinate and abscissa are chosen as shown because of the similarity with
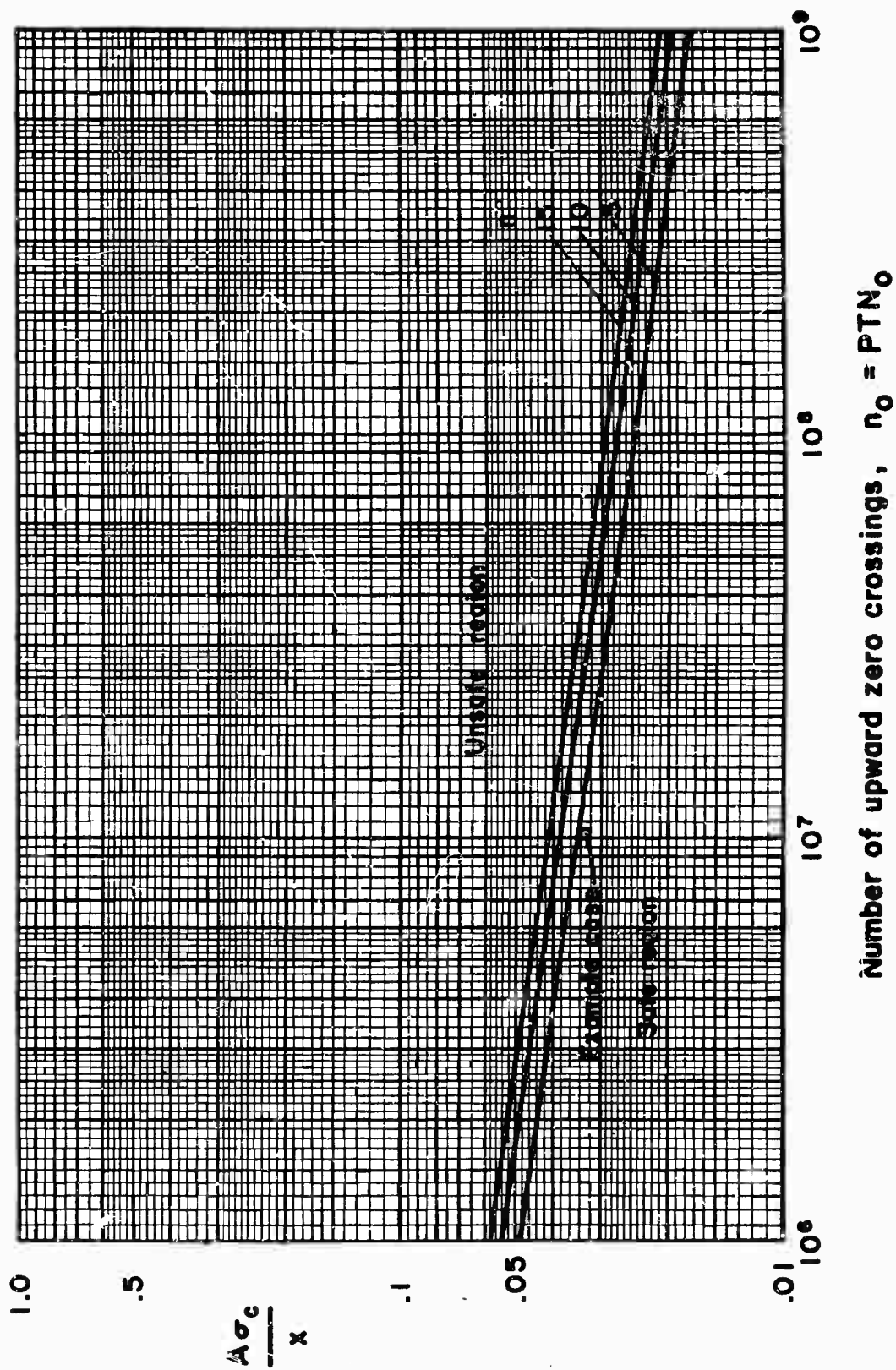
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Figure 2. Master Design Chart for Load Exceedance and Fatigue Life.

the familiar S-N plots as used in fatigue studies (see discussion on fatigue in Volume I). For design in this procedure, σ_c, P, T, and n are simply stipulated constant values, as for example $\sigma_c = 3$ fps, P = .10, T = 30,000 hrs, n = 5. Hence a convenient designation number indicating the basic gust design characteristics of a given airplane can be established by this procedure; for example, using the example numbers just quoted, a 3-3000 designation airplane is indicated, where 3 refers to σ_c, and where 3000 refers to the product of PT and hence denotes the total number of hours that the airplane is expected to be in turbulence in its lifetime. (The 5 is suppressed being chosen the same for all aircraft.) This procedure is very much akin to the discrete gust approach in which a design gust velocity U_{de} of 50 fps, for example, is stipulated. The procedure has considerable merit due to its simplicity, yet it still preserves most of the basic ingredients of the problem.

2.2. DESIGN BASED ON MISSION σ_c, P, AND T (Presently recommended to study the effect of extended flight times in severe turbulence; more data along the lines indicated are needed to place this procedure on a firm basis.)

In this procedure, explicit consideration is given to the various turbulence intensity levels encountered and to the amount of time spent in turbulence at various altitudes.

Figure 3 shows some tentative initial variations of σ_c and P that are estimated from the consideration of existing data and analysis. Note that only single measures of intensity and proportion of time are suggested here; that is, σ_c and P are not
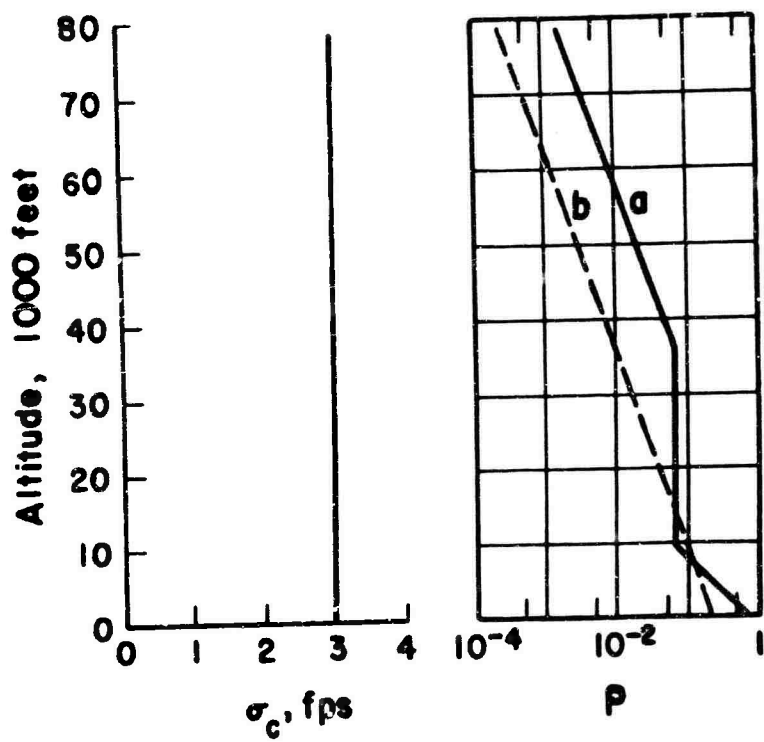
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Figure 3. Tentative σ_c and P Values.

broken up into severity categories as has been done in some past studies. Most of the data indicate the σ_c is nearly constant with altitude and in the neighborhood of 3 to 3.5 fps; a constant value of 3.0 has been chosen here. For P the choice is more in doubt; some results sugge.. curve a, others curve b. (The circumstance is quite odd, since of all quantities P should be the easiest to establish.) At the moment, curve a is felt to be more appropriate, although proof of this feeling is lacking. The recommendation follows that the curves of Figure 3 should be updated from the data that exist and from those being gathered.

The effective value for P applying to chosen missions, or to flight paths which are followed on the average, are found from the P variation curve by appropriately weighing the times at each altitude bracket. Figure 4 shows example results for the assumed flight plan shown also in the figure, using curve b of Figure 3. The 300 ft/min climb and descent rate is assumed to be an average value which takes into account times spent in holding patterns.

With values such as are given by Figures 3 and 4, design then proceeds as in the first procedure by use of Figure 2. An illustrative example is given to indicate the approach:

Design Conditions Given

$x = 28,000$ psi
$T = 30,000$ hrs $= 10.8 \times 10^7$ sec
$n = 5$

For The Mission

$\sigma_c = 3$ fps
$T_F = 3$ hrs
$h = 20,000$ ft $\Big\}$ $P = .085$ from Figure 4

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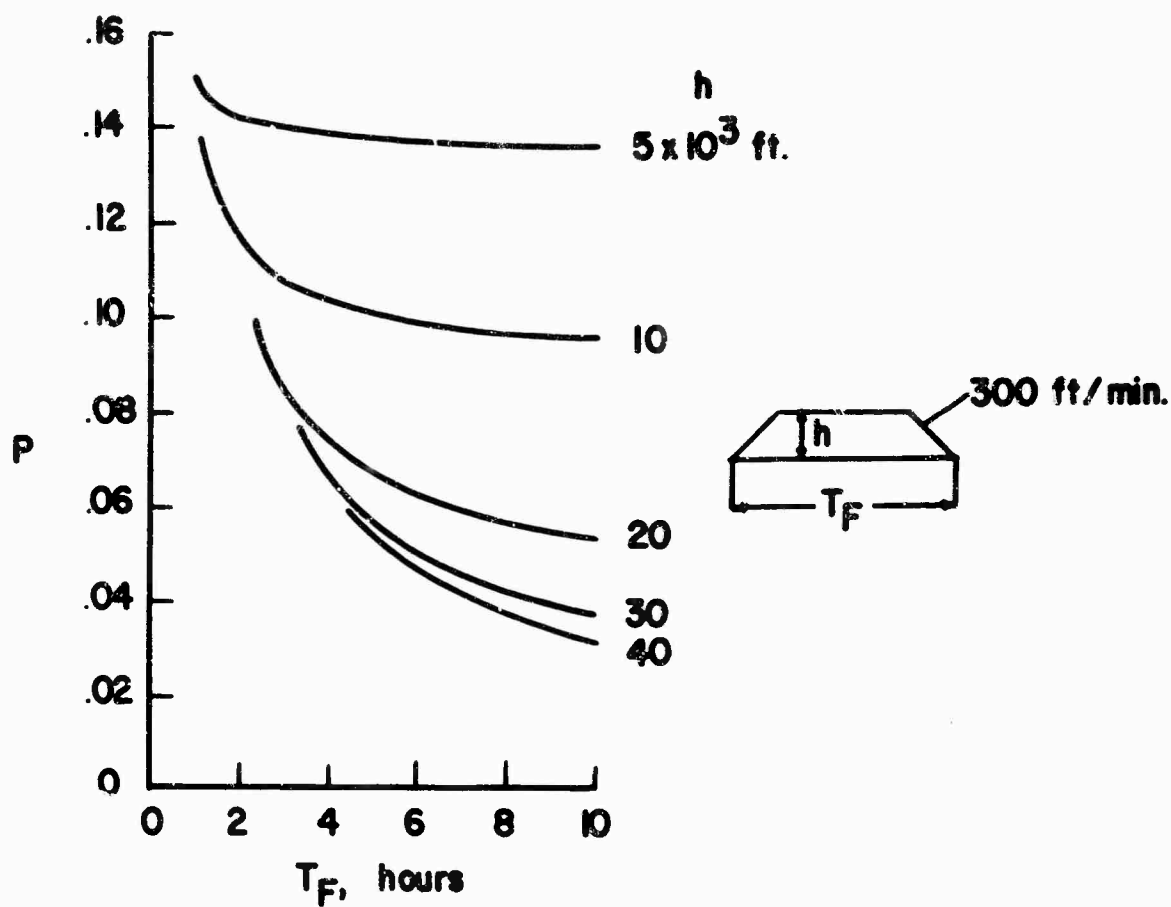
Figure 4. Proportion of Time in Turbulence for a Given Mission.

From The Airplane Preliminary Design

$A = 310$ psi/fps

$N_0 = 1.1$ cps

These values yield

$$\frac{A\sigma_c}{x} = .0332$$

$$PTN_0 = 10.1 \times 10^6$$

which plots on Figure 2 just below the $n = 5$ line as shown by the example point; a safe design is thus indicated.

For special purpose missions, such as low altitude flight in a high percentage of severe type turbulence, the values of σ_c and P must be selected accordingly. Figure 5 illustrates how σ_c might increase when operating under more severe turbulence conditions. If, for example, flights are involved with $P_2/P_1 = .05$, $\sigma_1 = 3$ and $\sigma_2 = 7.5$, then the σ_c for design use is found to be 3.72.

2.3. DESIGN BASED ON COMPARISON (Recommended as a useful adjunct for designs which bear close resemblance to proven gust critical airplanes)

Write Equation (2) for airplane 2, and also for airplane 1, and divide the resulting equations; the result is

$$\frac{(PTN_0)_2}{(PTN_0)_1} = \left[\frac{\left(\frac{A\sigma_c}{x}\right)_2}{\left(\frac{A\sigma_c}{x}\right)_1}\right]^{-6.74}$$
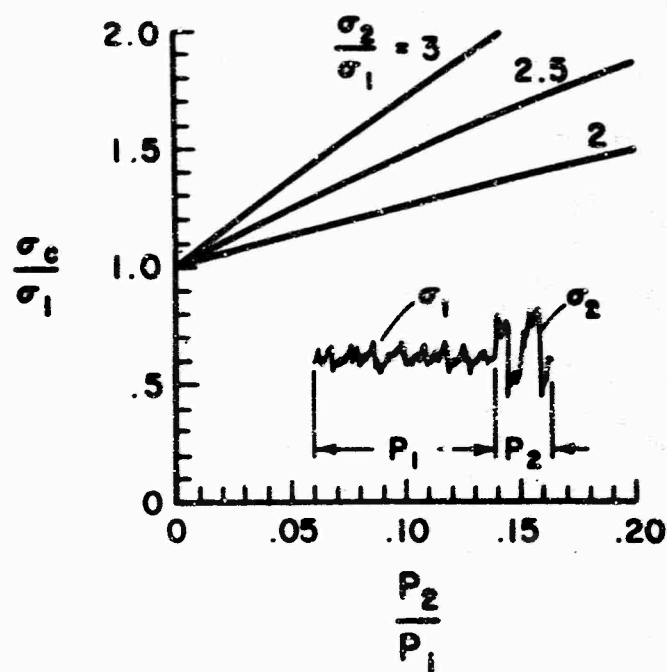
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Figure 5. σ_c Increase Due to Extended Severe Turbulence Encounter.

which plots as shown in Figure 6. Suppose that airplane 1 is a proven gust-critical airplane, and let 2 be the new airplane design. The ratios of the PTN_0 and $A\sigma_c/x$ quantities are established and if the resulting point plots on or below the curve of Figure 6 the new airplane is judged safe. This approach is quite attractive and has the merit of simplicity. Another advantage is the use of ratios which tends to minimize errors or uncertainties that may be common in corresponding parameters, such as in the choice of lift curve slope in the determination of the A's.

2.4. DESIGN BASED ON N_0 VERSUS x/A (Presently recommended as the first step toward improved design procedures)

In this approach, a further simplification to the first approach discussed, the values of P, T, and σ_c are chosen to be the same for all aircraft, and n is specified to be equal to or less than a certain number. On this basis, Equation (2) indicates the following very simple functional relation for safe design

$$N_0 \leq C(\frac{x}{A})^{6.74} \tag{3}$$

To establish the constant C, one way is simply to assume values of P, T, σ_c and n. Another way is an empirical approach; thus, suppose values of N_0 and x/A are computed for a number of existing gust critical aircraft which have proven themselves airworthy through many years of successful operation. These values when plotted would lead to the shotgun type pattern shown in Figure 7. A border of the type indicated by Equation (3) may then
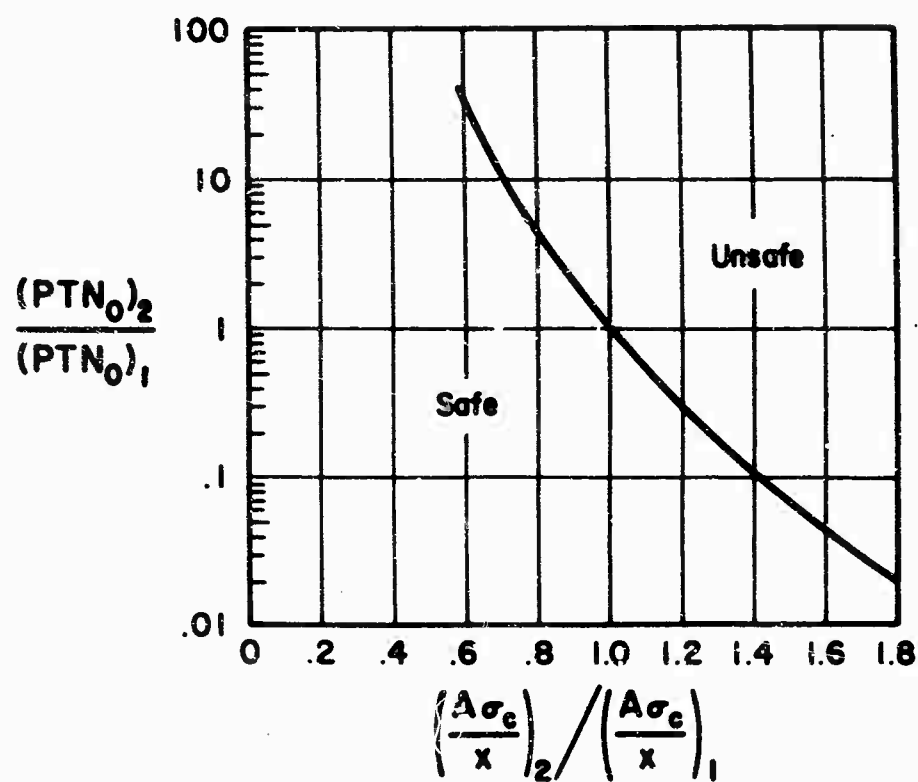
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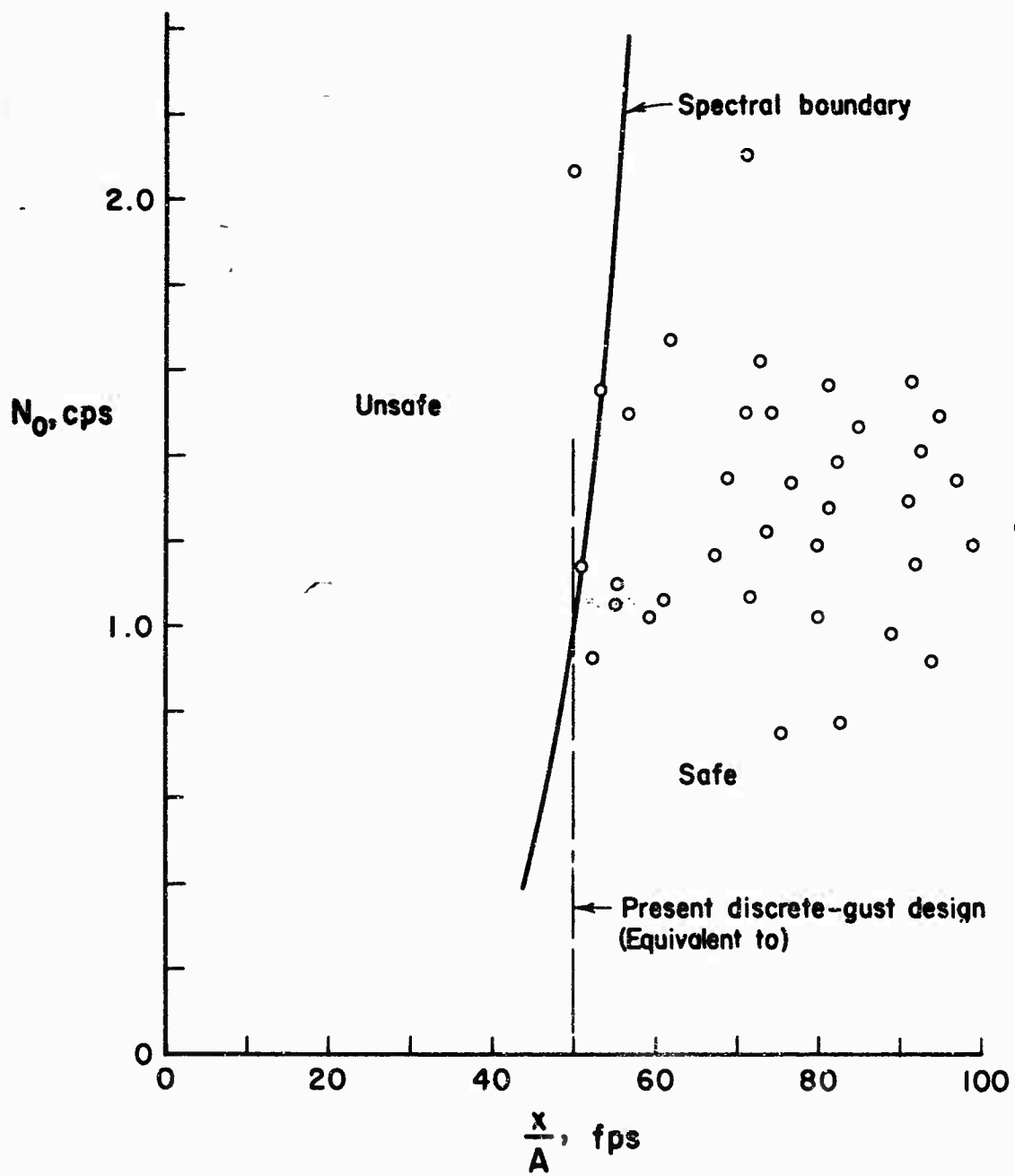Figure 6. Gust Design by Comparison (Note, stresswise,
adding metal lowers A).

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Figure 7. N_O Versus x/A Design Approach (Points and boundary are illustrative)

be drawn along the leftmost points as shown, with the inference that design points falling below this line represent safe design.

The points and line shown in Figure 7 are illustrative in nature and do not represent any specific computed values. However, this approach is the basis for the associated calculations that are being made for the Air Force by some of the aircraft companies. These calculated values of x/A and N_0 are to be used in helping to establish the definite position of a design border.

It is to be noted that all considerations of P, T, σ_c, and n are suppressed or eliminated in this procedure. Actually the procedure bears a close resemblance to the discrete gust design approach, but with two basic and noteworthy improvements. The first improvement is associated with the x/A axis. Values along this axis are the equivalent counterpart to U_{de}, the gust velocity value used in discrete gust design. To see this, consider

$$\sigma_x = A\sigma_w$$

and multiply both sides by η so that $\eta\sigma_x$ becomes equal to x_d, the design value of x, then

$$\frac{d}{A} = \eta\sigma_w = U_d$$

By contrast, a similar relation by the discrete gust design approach would appear

$$\frac{x_d}{A_{de}} = U_{de}$$

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indicating that U_d is similar in concept to U_{de}. The discrete gust design procedure is thus represented in Figure 7 by a fixed vertical line, as indicated. The newer approach discussed here not only allows a variation in U_d but establishes the value more accurately since A is determined by the more refined spectral relations compared to the establishment of A_{de}, which is based primarily on the K_g curve.

The second improvement is that of the N_O axis. In the discrete gust approach, no account is taken explicitly of the number of times gusts load levels are encountered. The N_O axis effectively takes this fact into account explicitly. Thus, in short, the boundary design curve of Figure 7 reflects that a tradeoff between the A and N_O value is possible in design; specifically, for example, increasing N_O must be accompanied by decreasing A.

In Section 5, two rigid body treatments are given, one along spectral lines and one from a discrete gust point of view. A comparison of results shows what terms in a spectral approach are equivalent to terms commonly used in the discrete gust approach. The material of Section 5 is to be used in the continuing study as a possible further means to help establish the position of the design boundary line on Figure 7.

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SECTION 3

GUST SPECTRA AND L VALUES

The spectra advocated here is that associated with isotropic turbulence and due to von Kármán, specifically

$$\phi(L\Omega) = \frac{\sigma_w^2}{\pi} \frac{1 + \frac{8}{3}(1.339 \ L\Omega)^2}{\left[1 + (1.339 \ L\Omega)^2\right]^{\frac{11}{6}}} \tag{4}$$

where σ_w is the r.m.s. value of gust velocity, and L is the turbulence scale. A plot of the equation is given in Figure 8, and an associated plot of $L\Omega\phi$ is given in Figure 9. This latter figure is significant in that it shows the contribution of the various frequency components to the mean square value σ^2; to see this, write

$$\sigma^2 = \int_0^\infty \phi(L\Omega)\,d(L\Omega)$$

$$= \int_0^\infty L\Omega\phi(L\Omega)\,dz$$

where $dz = d(L\Omega)/L\Omega$, or $z = \log L\Omega$. Thus with a semi-log plot with $L\Omega$ the log scale, the product $L\Omega\phi$ becomes a distribution curve indicating the distribution of σ^2 to the various frequency components.

The value of L to be used in design is at present uncertain, with estimates ranging from 500 to 5,000 feet. A quantity that has
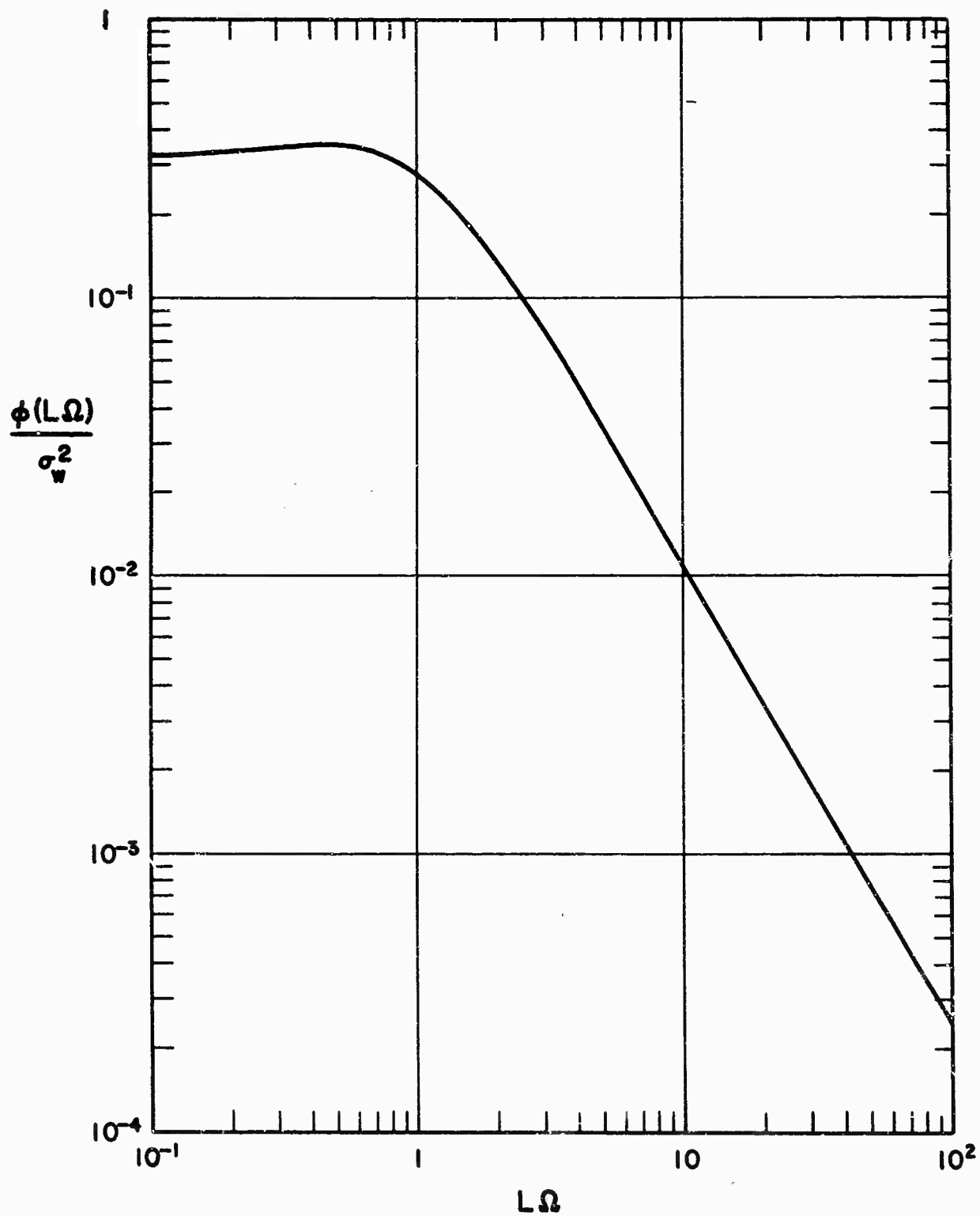
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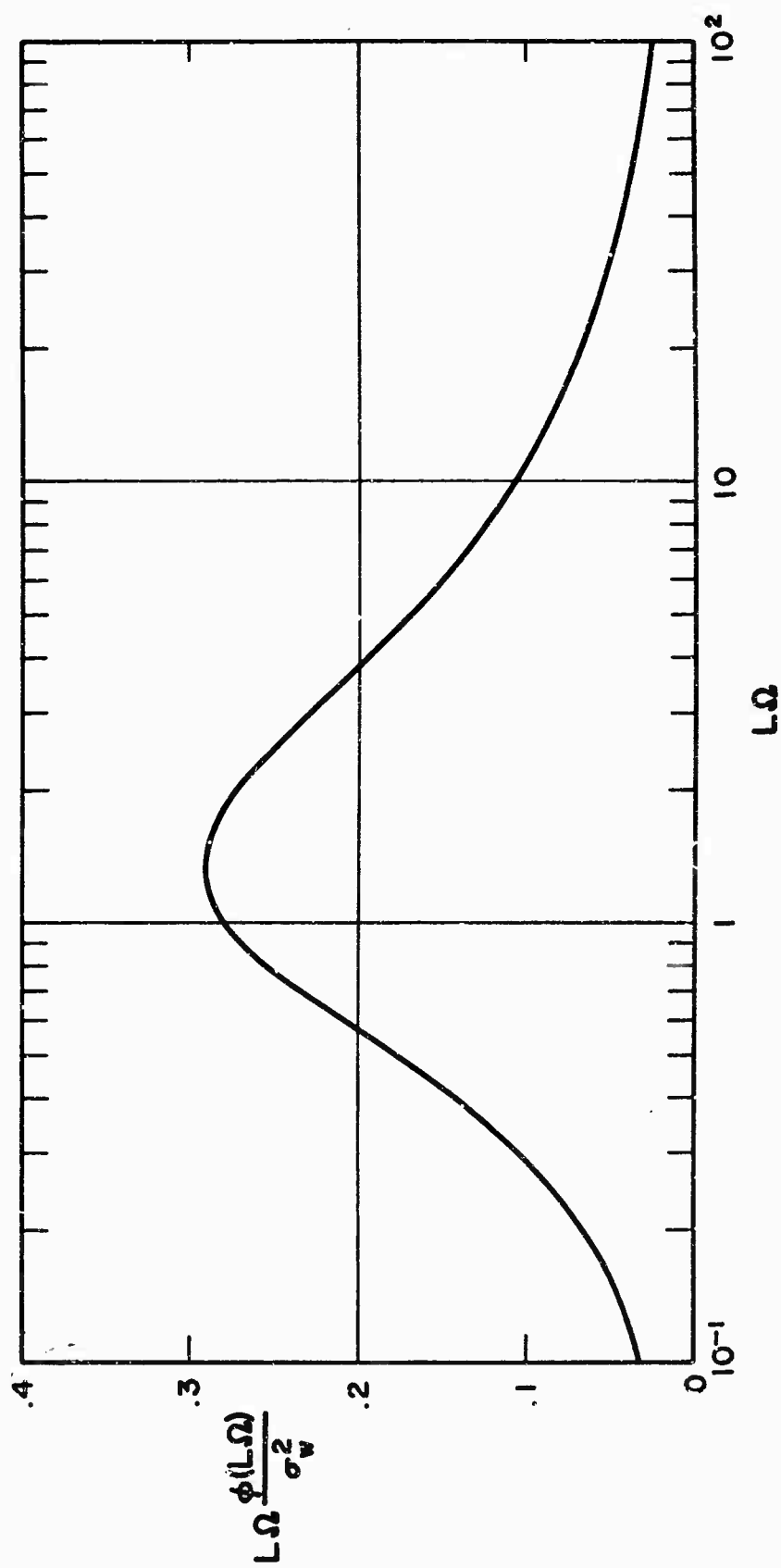Figure 8.   Spectral Function due to von Kármán.

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Figure 9. Distribution of σ^2 with Frequency.

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much significance in establishing L is the truncated value of
r.m.s. gust velocity defined by

$$\sigma_1^2 = \int\limits_{(L\Omega)_1}^{\infty} \phi(L\Omega)d(L\Omega) \tag{5}$$

The insertion of Equation (4) into this integral leads to the
results shown in Figure 10. In application to flight data, values
of σ_1, σ_w, and L must b. .uch as to satisfy the curve of
Figure 10. In general, σ_1 is established with good accuracy
but σ_w is questionable because of the presence of uncertain long-
wavelength components in the gust velocity records. Thus since
σ_1/σ_w is uncertain, the deduced L is also. Stated in other
terms, for a given σ_1, there can be many combinations of σ_w
and L which when substituted in Equation (4) give ϕ curves
which fit the flight data equally well. This fact can be demon-
strated in specific terms as follows. For Equation (4), the
result obtained from Equation (5) for $(L\Omega)_1 > 3$ is

$$\sigma_1^2 = \frac{.782 \ \sigma_w^2}{(L\Omega)_1^{2/3}} \tag{6}$$

With this equation and the fact that $\phi(\Omega) = L\phi(L\Omega)$, Equation (4)
may be rewritten

$$\phi(\Omega) = \frac{\sigma_1^2(\Omega_1)^{2/3}}{.782 \ \pi} \frac{L^{5/3}\left[1 + \frac{8}{3} (1.339 \ L\Omega)^2\right]}{\left[1 + (1.339 \ L\Omega)^2\right]^{\frac{11}{6}}} \tag{7}$$
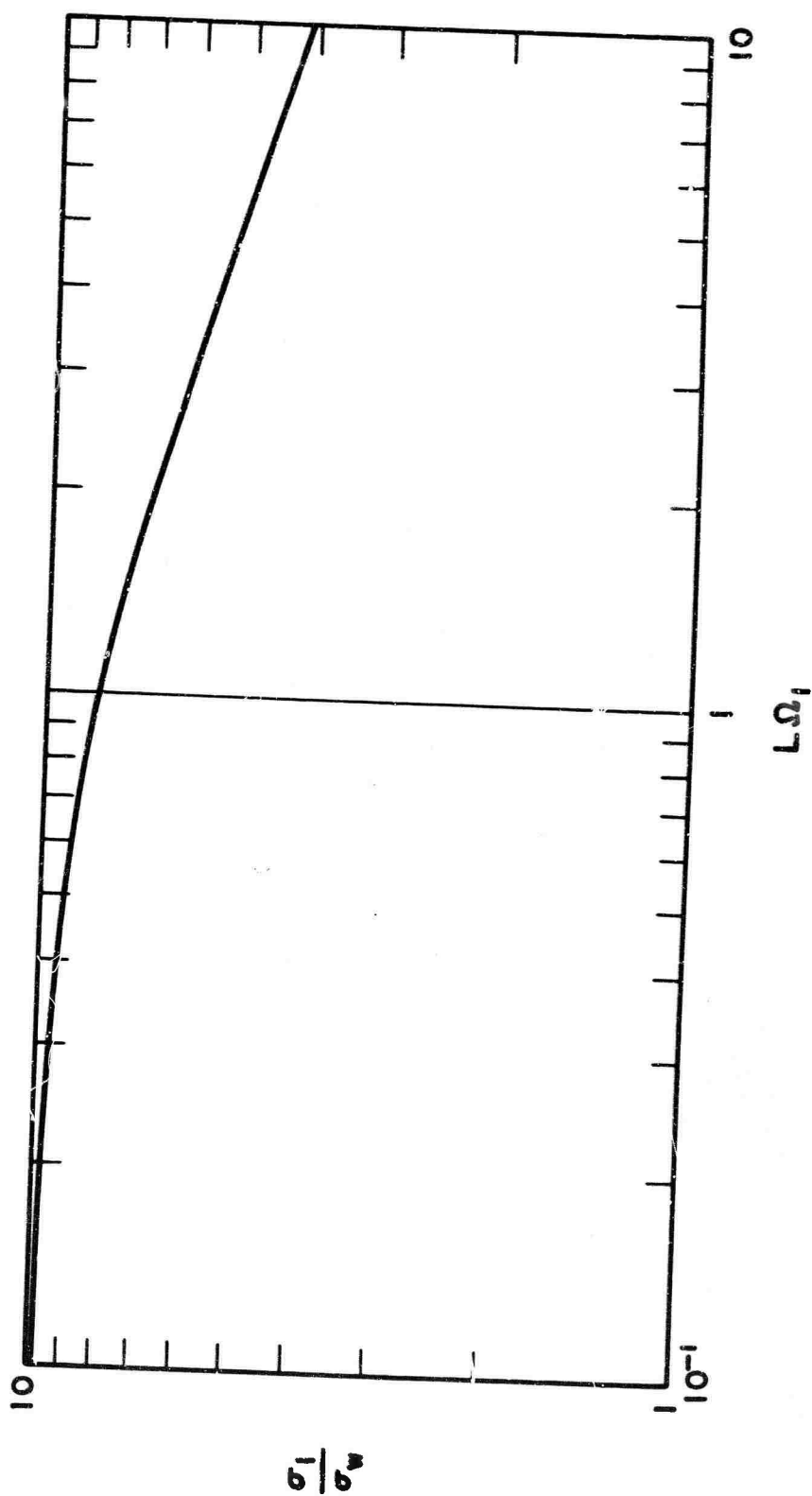
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Figure 10. Truncated r.m.s. Value σ_1 .

With Ω_1 arbitrarily chosen as .003, this equation yields the results shown in Figure 11. All curves have unit area beyond Ω = .003 regardless of ; or, if ϕ is plotted against Ω, all curves would yield the same truncated value σ_1^2. Appreciable difference is noted in the level of the curves at lower Ω, however, depending on L. This lower range is the range where evaluation of flight test data is difficult or impossible at present.

The use of ϕ curves in the form shown by Figure 11 may become useful and appropriate. In general, the value of A is found to depend strongly on the value of L used, see chapter on A and N_0 evaluation. However, if the ϕ curves of Figure 11 are used, then A_1, the counterpart to A, and as defined by

$$\sigma_x = A\sigma_w = A_1\sigma_1$$

is found to be practically insensitive to L. Thus, the use of a spectral curve with a specified truncated value may prove to be a worthwhile direction to go, since it minimizes the need for establishing and designating a design value of L. Possible methods of improving the ability to deduce L are being developed, however, in the continuing research study.
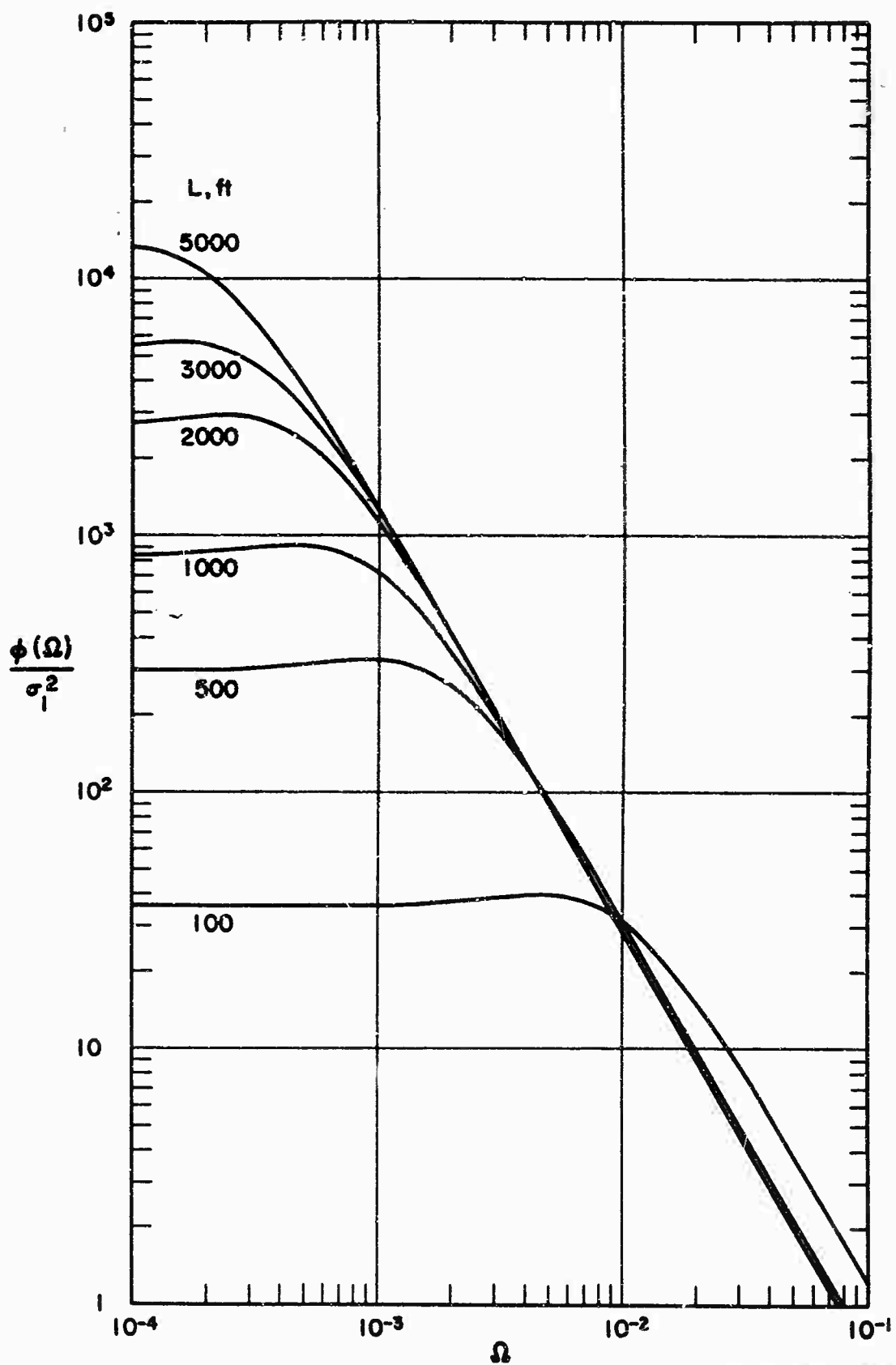
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Figure 11. Spectral Plots Having Equal Truncated Values σ_1 Beyond $\Omega = .003$

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SECTION 4

A AND N_O DETERMINATION

The basic input-output spectral relation is

$$\phi_X(\omega) = |H(\omega)|^2 \phi(\omega) \tag{8}$$

where $\omega = V\Omega$. With this, it is desired to establish A and N_O as defined by the following relations

$$\sigma_X = A\sigma_W \tag{9}$$

$$A = \frac{1}{\sigma_W} \left[\int_0^{\omega_c} \phi_X(\omega)\, d\omega \right]^{\frac{1}{2}} \tag{10}$$

$$N_O = \frac{1}{2\pi} \left[\frac{\int_0^{\omega_c} \omega^2 \phi_X(\omega)\, d\omega}{\int_0^{\omega_c} \phi_X(\omega)\, d\omega} \right]^{\frac{1}{2}} \tag{11}$$

Two questions arise; what value of L should be used in the equation for ϕ, Equation (4), and what should ω_c be. The scale L was discussed in a brief way in Section 3; some additional observations are made here. Calculation studies show that, in general, A depends on L (ω_c fixed). Typical results are shown in Figure 12. As suggested in Section 3, the use of a truncated value σ_1 of r.m.s. gust velocity might have merit. Thus, use Equation (6) and

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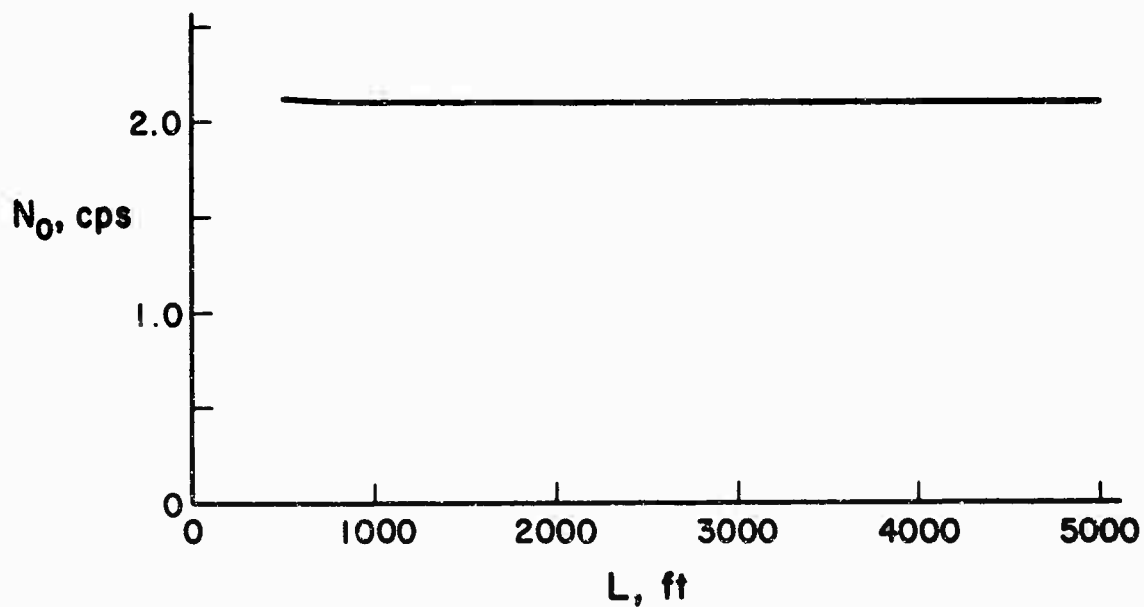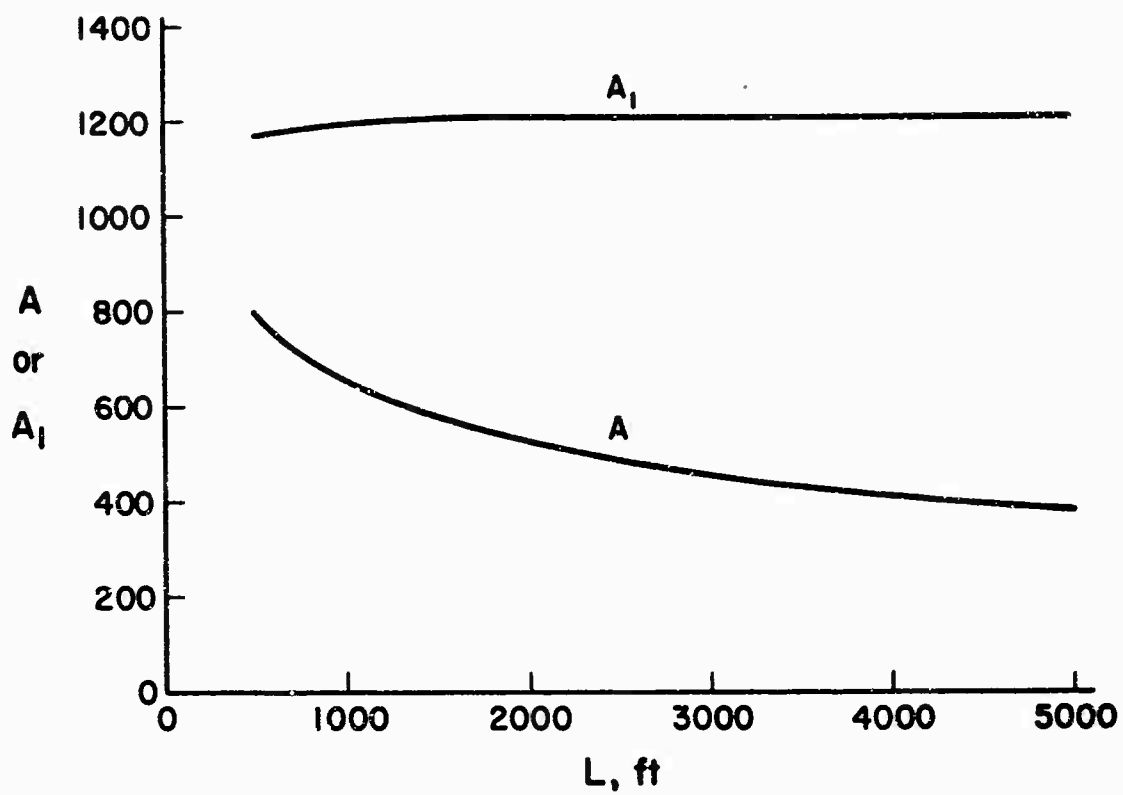
Figure 12. Example A, A_1, and N_0 Variations with (A_1 established for $\Omega_1 = .003$).

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rewrite Equation (9) as

$$\sigma_x = A\left(\frac{L\Omega_1}{.782}\right)^{\frac{1}{3}}\sigma_1$$

$$= A_1\sigma_1$$

The A_1 values for the A values shown in Figure 12 are also shown in the figure. It is seen that A_1 is nearly independent of L.

N_0 is found to be insensitive to L , Figure 12. However, N_0 is found to depend on ω_c in some cases. Typical variations of A and N_0 with ω_c are shown in Figure 13. A is found to level off nicely; however, N_0 may level off, case (a), or in some cases may continue to grow with increasing ω_c , case (b). For these later cases, some means of establishing a practical cutoff frequency is needed. A rule of thumb advanced here is to choose ω_c at the place where A has leveled off quite well and to read off N_0 at this frequency.

Other means for establishing ω_c are being studied.
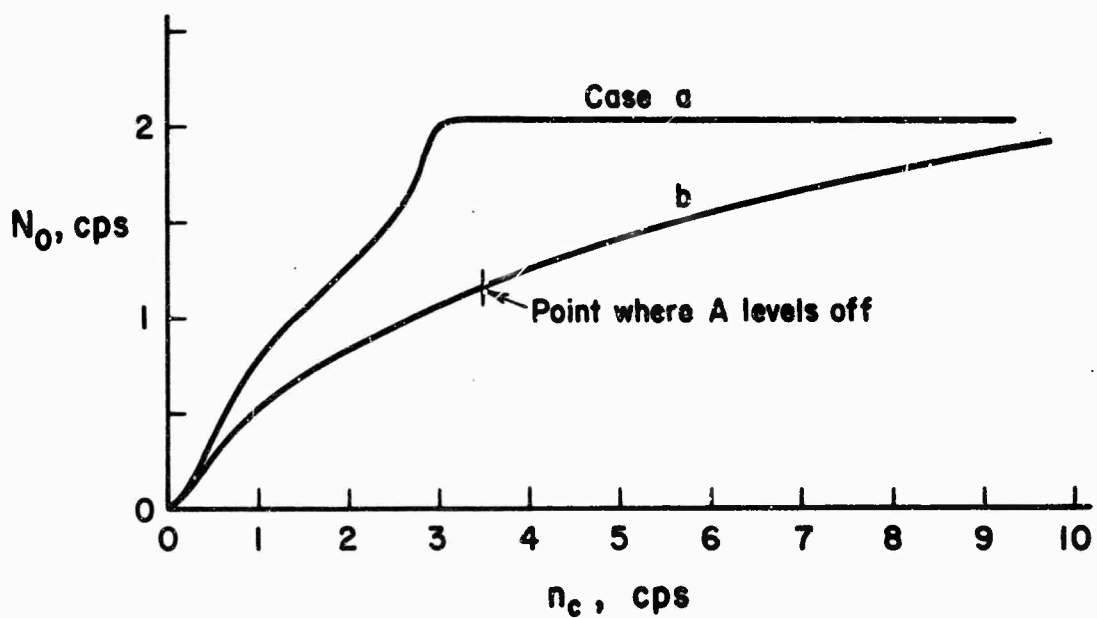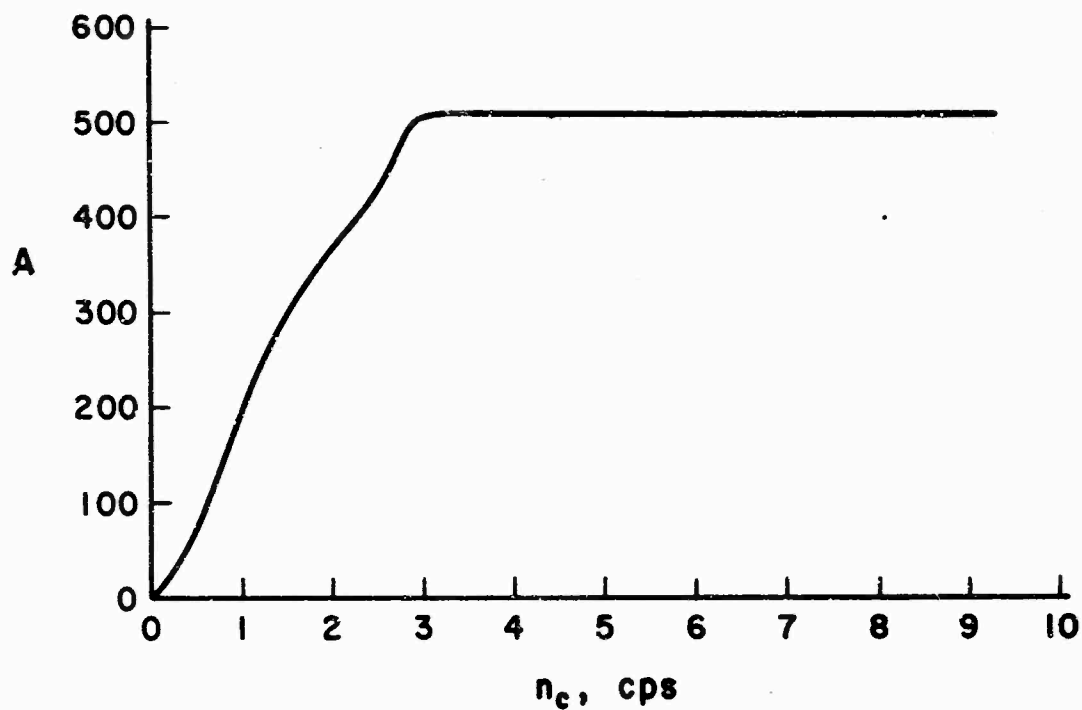
Figure 13. Illustrative A and N_0 Variations with n_c .

SECTION 5

SPECTRAL DESIGN LIMITS BASED ON DISCRETE GUST DESIGN

A rigid airplane with a single degree of freedom in vertical motion is considered here. Gust design by the discrete gust approach and also by a spectral approach are considered. Then by comparison of results, spectral design values are deduced from the discrete gust design values that have been used in the past.

The incremental acceleration in g units of the rigid body due to discrete gust encounter is given by

$$\Delta n = \frac{a\rho_0 S V_e}{2W} U_{de} K_g \qquad (12)$$

where K_g is the gust alleviation factor which depends on $\mu_g = 2W/a\rho c g S$, the airplane mass parameter (see Reference 2).

From Reference 3, it may be shown that the transfer function $|H(\omega)|^2$ for vertical acceleration is given by

$$|H_{\Delta n}|^2 = \left(\frac{4V}{c_0 g \mu_0}\right)^2 \frac{k^2}{\left(\frac{2F}{\mu_0}\right)^2 + \left(k + \frac{2G}{\mu_0}\right)^2} \cdot \frac{1}{1 + 2\pi k}$$

where $\mu_0 = 4\mu_g$. For use here, the spectral Equation (4) may be put in the form

$$\phi(k) = \frac{2L}{c} \phi(L\Omega) = \frac{\sigma^2}{\pi\left(\frac{2L}{c}\right)^{\frac{2}{3}}} \frac{\left(\frac{2L}{c}\right)^{\frac{5}{3}} \left[1 + \frac{8}{3}(1.339 \frac{2L}{c} k)^2\right]}{\left[1 + (1.339 \frac{2L}{c} k)^2\right]^{\frac{11}{6}}}$$

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Note that care has been exercised here to put H and ϕ in forms so as to make subsequent results condense as much as possible into single nondimensional type curves rather than into a family of curves. The spectral equation for Δn is then

$$\phi_{\Delta n}(k) = |H_{\Delta n}(k)|^2 \phi(k)$$

Values of A and N_0 as found from Equations (10) and (11) were found to be

$$A = \frac{a\rho SV}{2W} \left(\frac{c}{2L}\right)^{\frac{1}{3}} B \tag{13}$$

$$N_0 = \frac{2V}{c} N_{01} \tag{14}$$

where B and N_{01} are shown in Figure 14. The near invariance of B and N_{01} to L/c is to be noted.

With the parameter $x/A\sigma_c$ as discussed in Section 2 in mind, let x be Δn; then Equation (12) divided by $A\sigma_c$ gives

$$\frac{\Delta n}{A\sigma_c} = \frac{\rho_0 V_e U_{de}}{\rho V \sigma_c} \left(\frac{2L}{c}\right)^{\frac{1}{3}} \frac{K_g}{B} \tag{15}$$

or using Equation (6) (with $\sigma_w = \sigma_c$)

$$\frac{\Delta n}{A\sigma_c} = \frac{\rho_0 V_e U_{de}}{\rho V \sigma_1} \frac{.885}{(k_1)^{\frac{1}{3}}} \frac{K_g}{B} \tag{16}$$

where, in analogy to the discussion of σ_1 given in Sections 3 and 4, σ_1 here refers to the truncated r.m.s. value associated

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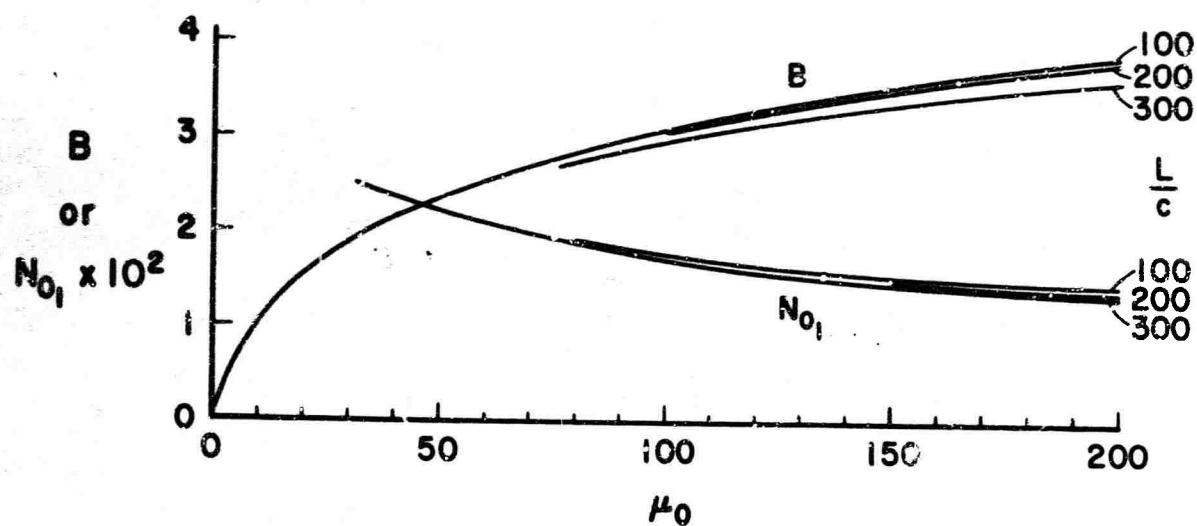Figure 14. B and N_{O1} Curves.

$$\frac{Kg}{B} = \frac{\rho V}{\rho_0 V_0}\left(\frac{c}{2L}\right)^{\frac{1}{3}}\frac{1}{U_{de}}\frac{\Delta n}{A}$$
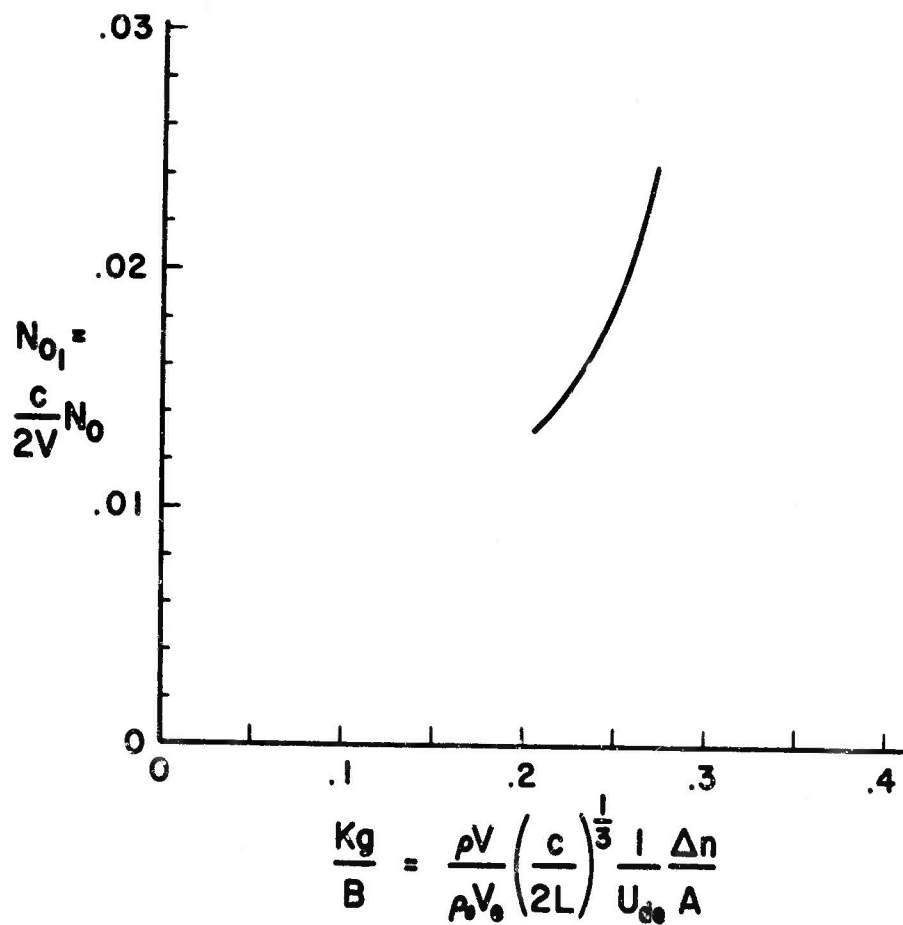
Figure 15. N_{O1} Versus $\Delta n/A$ as Based On Combined Discrete-Gust and Spectral Analysis.

with the area under the $\phi(k)$ spectrum beyond k_1. From Figure 14 and the K_g curve of Reference 2, the curve shown in Figure 15 follows. The close similarity of this curve with Figure 7 is noted. Thus, the results shown in Figure 15, based solely on rigid body response considerations, but which considerations have formed the basis for gust design for many years, may prove quite helpful in establishing the design boundary curve of Figure 7. As an illustrative indication of possible limits on x/A and N_0, go back to Equations (14) and (15). Suppose $\rho = \rho_0$, $V = V_e$, take $U_{de} = 50$, and choose $K_g/B = .26$ and $N_{01} = .021$ as representative values; Equation (15) indicates the following

$\dfrac{2L}{c}$	$\dfrac{\Delta n}{A}$	$\dfrac{\Delta n}{A\sigma_c}$ (for $\sigma_c = 3$)
100	50.2	20.1
200	76.1	25.4
300	87.2	29.1
400	95.7	31.9

while Equation (14) yields

$\dfrac{2V}{c}$	N_0
20	.42
30	.63
40	.84
50	1.05

The values of $\Delta n/A$ and N_0 are noted to confirm the significant design range shown in Figure 7, and the $\Delta n/A\sigma_c$ values are seen to fall in the upper or design range of Figure 1. The importance of knowing realistic values of $2L/c$ and σ_c is brought out, however,

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alternatively the importance of knowing the truncated value σ_1 as used in Equation (16) is shown.

SECTION 6

CONCLUSIONS AND RECOMMENDATIONS

Most of the conclusions and observations arrived at in the
course of study were brought out in the development in each
chapter. A summary of some of the more significant points as
well as others not specifically covered is given here.

The study has resulted in four possible design procedures of
differing complexity and usefulness, all of which look good.
Appropriate values of input design parameters, σ_c, P, T, must
now be established. Some of these evaluations must await better
flight data, or at least flight data reduced in a more appropriate
way. The form and shape of the universal curve needs verification.
Convenient analytical expressions for the universal curve are
being studied, and consideration is being given simultaneously to
establishing the associated and significant frequency distribution
curves for the r.m.s. value of gust velocity σ.

At present, much of the flight data are conflicting and much
cannot be used to advantage because the analysis was not done in
a proper or standardized way or because the appropriate quantities
are not presented. With respect to establishing a universal load
exceedance curve, some flight data indicate that the curve is
concave upward (on a log-log plot), as used herein, while other
data indicate the curve might be straight or even concave downward.
The differences noted are believed due primarily to differences in
the way various investigators take, interpret, process, and present
flight data. Thus, a more consistent data gathering and analysis

procedure is highly desirable. To this end, consideration is being given to the way flight data ought to be taken and analyzed to yield results in general, or "universal" form, and convenient for design application use.

The appropriate value or values of L and σ_c still needs to be found. Better means for analyzing flight data in a meaningful way are being studied with the aim of establishing realistic values of L and σ_c. Likewise, the determination of N_0 still presents some problems. Methods to place this evaluation on a firm basis are also being pursued.

Other related work, such as showing how N_0 is related to the initial curvature of the autocorrelation function, or deducing turbulence velocities from mean wind profiles, or of combining spectral shapes to alter the over-all form, etc., is continuing.

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13. ABSTRACT

A development is given of four possible procedures for designing aircraft for gusts based on power spectral techniques. Each is of a different level of complexity but all are attractive at the moment. In one, explicit consideration of atmospheric turbulence makeup is bypassed altogether. Another makes use of simple over-all composite values of rms value of turbulence intensity and proportion of time in turbulence. A third is more detailed and considers mission aspects explicitly. A fourth is a rather simple comparison approach. Items which represent areas of uncertainty are brought out, chief among these being the turbulence scale value, and the practical means for establishing cutoff values in the determination of the basic frequency parameter N_0 of the airplane. The need for a more unified and consistent data gathering program is brought out, among various agencies and countries, and from the standpoint of operational as well as research flights. Analogous rigid body treatments by spectral and discrete-gust lines are made and compared so that the discrete-gust design levels could be used as a guide to help establish input number by a spectral approach. The material of the report forms the basis for the next phase of study which has the aim of developing one or more of the procedures into a firm design language. Computational data from existing gust critical aircraft will help guide number or design boundary selection in this task.

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